Anderson transition in one-dimensional systems with spatial disorder

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Abstract

A simple Kronig-Penney model for one-dimensional (1D) mesoscopic systems with δ peak potentials is used to study numerically the influence of a spatial disorder on the conductance fluctuations and distribution at different regimes. We use the Lévy laws to investigate the statistical properties of the eigenstates. We found the possibility of an Anderson transition even in 1D meaning that the disorder can also provide constructive quantum interferences. We found at this transition that the conductance probability distribution has a system-size independent shape with large fluctuations in good agreement with previous works. In these 1D systems, the metallic phase is well characterized by a Gaussian conductance distribution. Indeed, the results for the conductance distribution are in good agreement with the previous works in 2D and 3D systems for other models.

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I. INTRODUCTION

The disorder-induced metal-insulator transition (MIT) has been studied extensively for decades [2, 3] and continues to attract much attention. Scaling theory [2] predicts that all eigenstates of noninteracting electrons are localized in one-dimensional (1D) and two-dimensional (2D) systems for any amount of disorder. It is commonly believed that MIT occurs only for dimensions $d > 2$ and the system remain insulator for $d < 2$ [4]. However, recently, it was suggested that a 2D Anderson model of localization with purely off-diagonal disorder might violate this general statement since non-localized states were found at the band center [5]. It was found that a transition present in this 2D model can be described by a localization length which diverges with a power-law behaviour. The possible existence of the Anderson transition in 2D systems without interaction and spin-orbit effects becomes recently a subject of controversy in the literature [6, 7]. More recently, Asada et al. [8] studied the β function that describes the scaling of the quantity Λ as:

$$\beta(Ln\ \Lambda) = \frac{dLn\ \Lambda}{dLn\ L} \tag{1}$$

where Λ is the ratio of the quasi-one-dimensional localization length to the system width for electrons on a long quasi-one-dimensional system of width L. They indicated the possibility of an Anderson transition for dimensions $d \leq 2$ in disordered systems of non-interacting electrons. On the other hand, strong numerical evidence of a mobility edge was found in disordered photonic systems in two dimensions [9]. Recently, surprising results were found in the properties of disordered graphene systems [10, 11, 12, 13].The latest studies, both theoretical and experimental, led to the amazing conclusion that there is no localization in disordered graphene, even in the one-dimensional (1D) situation.

It is well known that the conductance g is not a self-averaged quantity [14] and therefore fluctuates with the Fermi energy, chemical potential and sample size. Since the conductance does not obey the central limit theorem [15], it is necessary to investigate not only the first two moments but the entire probability distribution. Numerical results in 2D and 3D disordered systems showed that the conductance is normally distributed in the metallic regime while for strongly localized systems (insulating regime) a log-normal distribution was found [16]. The exact form of the probability distribution at the transition is not well

known. In such a regime, it was proven that the conductance distribution is independent of the microscopic details of the model (determined by the distribution of the disorder), of the system size and of the position of the critical point which separates the metallic and the localized regime in the phase space of external parameters (energy, disorder). The universality of the conductance distribution was studied and confirmed by Markos et al. for 2D and 3D models [17, 18]. The system-size invariance of the conductance distribution at the critical points of the MIT was confirmed for 3D and 4D systems in [19, 20, 21].

The aim of this paper is to investigate the possibility of an Anderson transition in 1D spatially disordered systems of noninteracting electron. Three regimes of electron transport are studied: the insulating regime corresponding to strong disorder, the metallic regime (corresponding to an infinitesimal disorder) and the transition regime. In the present work, Levy statistics [22] are used.

II. MODEL DESCRIPTION

We consider a Kronig-Penney model applied to a 1D system of delta potentials with random positions (spatial disorder) . The corresponding Schrödinger equation can be read:

$$\left\{ -\frac{d^2}{dx^2} + \sum_n \beta\ \delta(x - x_n) \right\} \Psi(x) = E\Psi(x) \tag{2}$$

Here $\Psi(x)$ is the single particle wave-function at x, β the potential strength and E the single particle energy in units of $\hbar^2/2m$ with m the electronic effective mass. The two ends of the system are assumed to be connected ohmically to ideal leads (where the electron moves freely)

The second order differential equation (2) can be mapped by means the Poincaré map representation [23]:

$$\Psi_{n+1} = \left[\cos(kl_{n+1}) + \frac{\sin(kl_{n+1})}{\sin(kl_n)} \cos(kl_n) + \beta_n \frac{\sin(kl_{n+1})}{k} \right] \Psi_n - \frac{\sin(kl_{n+1})}{\sin(kl_n)} \Psi_{n-1} \tag{3}$$

where Ψ_n is the value of the wave-function at site n and $k = \sqrt{E}$ is the electron wave number $l_{n+1} = x_{n+1} - x_n$ is the inter-atomic spacing. $l_{n+1} = a + s$, s being a random variable uniformly distributed as $-W/2 < s < W/2$ (W being the degree of the disorder).

For equally spaced potentials, the interatomic spacing $l_{n+1} = l_n = a$ ($W = 0$) and equation (2) reduces to that found by Soukoulis et al.[23]:

$$\Psi_{n+1} + \Psi_{n-1} = \left[2\cos\sqrt{E} + \beta_n \frac{\sin\sqrt{E}}{\sqrt{E}} \right] \Psi_n \tag{4}$$

The solution of equation (3) is carried out iteratively by taking the two initial wave functions at sites 1 and 2 : $\Psi_1 = \exp(-ik)$ and $\Psi_2 = \exp(-2ik)$. We consider here an electron having a wave number k incident at site $N+3$ from the right (by taking the chain length $L = N$, i.e. $N+1$ scatterers). The transmission coefficient (T) reads

$$T = \frac{|1 - exp(-2ikl_n)|^2}{|\Psi_{N+2} - \Psi_{N+3} exp(-ikl_n)|^2} \tag{5}$$

The dimensionless conductance ($g = \frac{G}{e^2/h}$) can be obtained from the transmission coefficient T via the Landauer formula for 1D systems [24]:

$$g = \frac{2T}{1-T} \tag{6}$$

where the factor two arises from the two possible states of the electron spin.

and the variance of conductance ($Ln(g)$) reads:

$$var(Ln(g)) = \ < Ln(g)^2 > - < Ln(g) >^2 \tag{7}$$

where $< ... >$ denotes an average over different realizations of the disorder.

III. RESULTS AND DISCUSSION

In this section, we discuss numerical results of the conductance probability distribution for different amount of spatial disorder of 1D mesoscopic systems of delta potentials. In order to obtain the probability distribution of the conductance, we build a statistical ensemble of 10^4 samples differing only in the realization of the disorder.

Firstly, we examine the effect of this kind of disorder on electronic eigenstates by calculating the transmission coefficient and the conductance probability distribution. Figure 1 shows the scaling of the transmission coefficient $< -Ln(T) >$ for different degrees of spatial disorder and for two values of electron energy $E = 4$ (corresponding to an energy in the band gap of the periodic system) and $E = 12$ (corresponding to an energy in the allowed

	$E=4$	$E=4$	$E=12$	$E=12$
W	ζ	γ	ζ	γ
0.005	4797.866	2.084×10^{-4}	1.7832	0.5607
0.1	1205.313	8.2966×10^{-4}	1.8082	0.5530
0.2	0301.204	0.00332	1.9205	0.52068

TABLE I: Calculated results of the Lyapunov exponent γ and the localization length ζ as a function of the disorder W.

band, see inset of Figure 1.a). For strong disorder the transmission coefficient decreases exponentially with the length scale. Furthermore, the localization length ζ and the Lyapunov exponent $\gamma = 1/\zeta$(slope of the curve) are deduced. The calculated results are presented in table I. Fig.1a shows that increasing disorder leads to strong localization (since the localization length ζ decreases with the disorder strength W). This is in good agreement with previous results with topological disorder i.e the disorder localizes the electronic eigenstates. However, this is not the case as shown in Fig.1b where ζ increases with the disorder. This behaviour was previously pointed out by Nimour et al. for 1D spatially disordered systems with finite width potentials [25]. More recently, it has been shown that the disorder could either suppress or enhance the transmission in disordered grapheme superlattices [26].

Since our system is disordered, its conductance depends on specific realizations of disorder (for a given L), it is then appropriate to study the whole probability distribution of conductance. For strong disorder where $w = 0.2$, we have $\zeta = 300.2$, which is smaller than the size of the system. The localization of the electronic eigenstates is confirmed in figure 2a where the probability distribution of the natural logarithm of the conductance ($Ln(g)$) is plotted. In this figure the probality distribution is Gaussian indicating the general behaviour in this regime [16]. This figure shows also that the mean conductance decreases as the system size increases. In a previous interesting paper, Vagner et al. [27] studied analytically electron transport in a one-dimensional wire with disorder modeled as a chain of randomly positioned scatterers with δ-shaped impurity potential (similar to our model). They found that the distribution $P(f)$ of the variable $f = ln(1+\rho)$ (ρ being the resistance)

has a non-Gaussian behaviour in the limit of weak disorder. This result is confirmed for the conductance in our system. It is clearly seen from figure 2b that when the disorder decreases, the conductance distribution shows a deviation from its log-normal form indicating delocalization of the eigenstates.

In order to further understand the behaviour of the conductance, we investigate the conductance distribution $P(g)$ for different amounts of spatial disorder and for different system sizes. All these distributions show long power law tails decreasing for large values of the conductance g (see Figure 3). Therefore, we use the Levy statistics $L_\mu(Z)$ of index μ which decreases as $Z^{-(1+\mu)}$ for large values of Z [22]. It is found that the conductance distribution $P(g)$ behaves as $g^{-(1+\mu)}$ for large values of g. The exponent μ is then extracted from the log-log plot of $P(g)$ for large values of g which is linear with a slope equal to $-(1+\mu)$ (see the inset). It is known that if $\mu > 2$, the probability distribution is normal. On the other hand, if $\mu < 2$ it means that the distribution is log-normal. In figure 4, the index μ is plotted as a function of degree of spatial disorder W and for different system size L. In this figure two distinguishable regions are shwn: a region of $\mu < 2$ (corresponding to an insulating regime) and a region of $\mu > 2$ (corresponding to a metallic regime for small disorder W). The intersection point of curve $\mu(W)$ with the strait line $\mu = 2$ corresponds to the metal-insulator transition region. The critical point separating the metallic and insulating regimes corresponds to a critical disorder with $W_c = 0.0035$. Figure 4 shows also that the index μ is independent of the system size at the transition.

Let us now examine the conductance probability distribution for different amounts of spatial disorder. In Figure 5 are plotted the probability distributions of $Ln(g)$ (Fig.5a) and g (Fig.5b) for different system size ($L = 500, 700, 800$ and 900) for the critical disorder $W_c = 0.0035$. The conductance distribution seems to be neither normal nor log-normal. The size independence of the conductance distribution is in agreement with the observed one for 2D and 3D systems [17, 19, 21] at the transition. The long tail of the distribution in Fig.5a is representative for large fluctuations in good agreement with the results of Shapiro [28] and Shapiro and Cohen [29] for the metal-insulator transition.

For an infinitesimal disorder $W < W_c$ corresponding to $\mu > 2$, where $w = 0.0025$, we have $\zeta = 47234$, which is much larger than the system size L. The conductance distribution has a Gaussian form indicating the metallic regime (Figure 6). This figure shows the general

behaviour for this regime. Once the system is away from the critical point, the conductance distribution $P(g)$ begins to show size dependence. Distribution $P(g)$ moves toward higher values of the conductance as the system size increases.

IV. CONCLUSION

We have used the Kronig-Penney model in a simple one-dimensional (1D) system with spatial disorder to determine the metal-insulator transition and examine the size independence of its distribution at this transition. To find the critical point, we have used stable Levy laws [22]. We found the possibility of an Anderson transition even in 1D meaning that the disorder can also provide constructive quantum interferences. We found the critical disorder W_c for this transition. Indeed, the results are in good agreement with the previous works in 2D and 3D systems for other models [17, 19, 21] for metal-insulator transition. The conductance probability distribution is found to be Gaussian in the metallic regime. At the transition, the distribution $P(lng)$ exhibits the typical asymmetric behaviour while in the localized regime, the distribution is log normal. It is important to study the universality of the conductance distribution at the transition in a system of finite width potentials where the conductance fluctuations are less important in comparison to the present model [30], and to find the critical points of the transition in the energy-disorder phase space and for different kinds of disorder. It is also important to investigate the effect of spatial disorder on conductance fluctuations in different regimes. These problems will be the subject of a forthcoming paper.

Acknowledgments

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Figure captions

Figure 1: Transmission coefficient <-Ln(T)> as a function of system size averaged over 1000 realizations of the same system for $V = 2$ and different amounts of spatial disorder W and for a) $E = 4$ (in the miniband). b) $E = 12$ (in the gap). Inset in a: Transmission Vs energy for a periodic system.

Figure 2: Probability distribution of $-log(g)$ for $E = 4$, $V = 2$, $L = 1600$ and for a) $W = 0, 2$. b) $W = 0, 0036$.

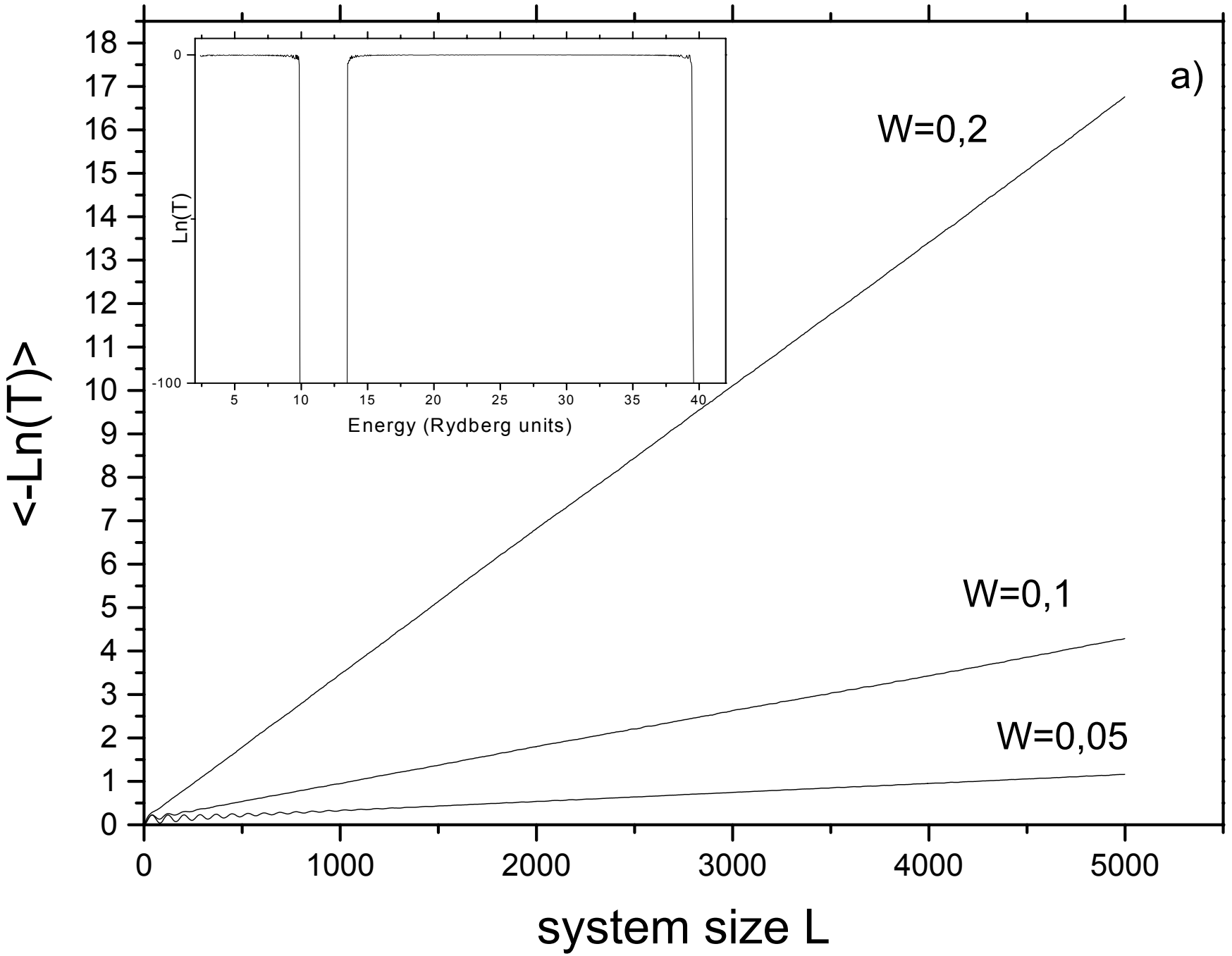
Figure 3: Conductance probability distribution for $L = 1500$, $E = 4$ and $W = 0.0034$. Inset: log-log plot of the tail of the distribution, the strait line corresponds to the best power-law fit to the tail $P(g) \sim g^{-(1+\mu)}$ with $\mu = 1.02$.

Figure 4: Lévy exponent μ as a function of disorder for $E = 4$, $V = 2$ and different system size L.

Figure 5: a) Probability distribution of the conductance g for $E = 4$ and $V = 2$. b) Probability distribution of $-\ln(g)$ compared with a Gaussian with the same mean and variance.

Figure 6: Conductance probability distribution in the metallic regime for different system size L with $W = 0.0025$, $E = 4$ and $V = 2$ compared with a Gaussian with the same mean and variance.

Figure 1a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W=0,1
W=0,05
<-Ln(T)>
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1000
2000
3000
4000
5000
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
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18
Ln(T)
0
-100
5
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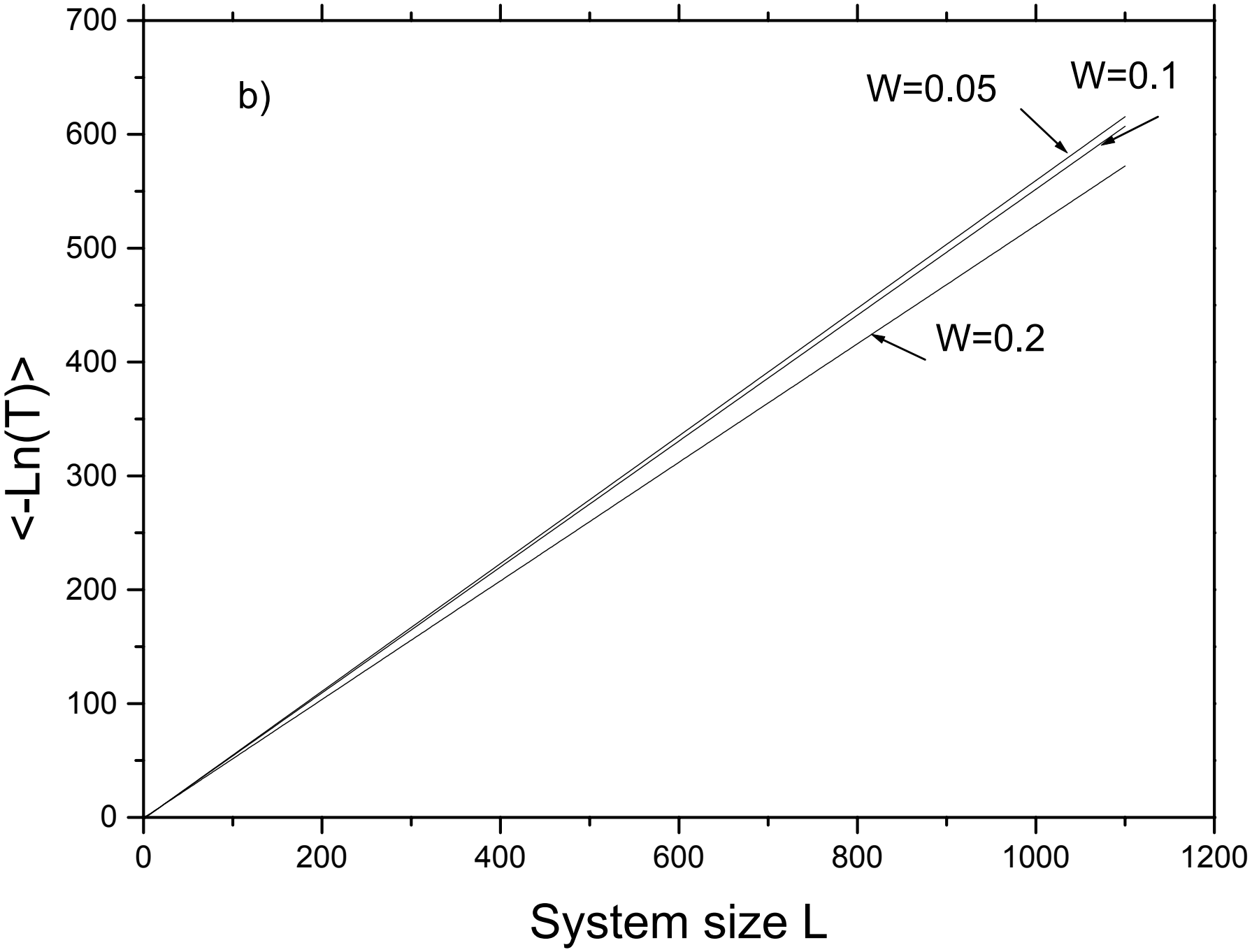
20
25
30
35
40
Energy (Rydberg units)

Figure 1b


b)
W=0.05
W=0.1
W=0.2
<-Ln(T)>
System size L
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200
300
400
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700
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400
600
800
1000
1200

Figure 2a

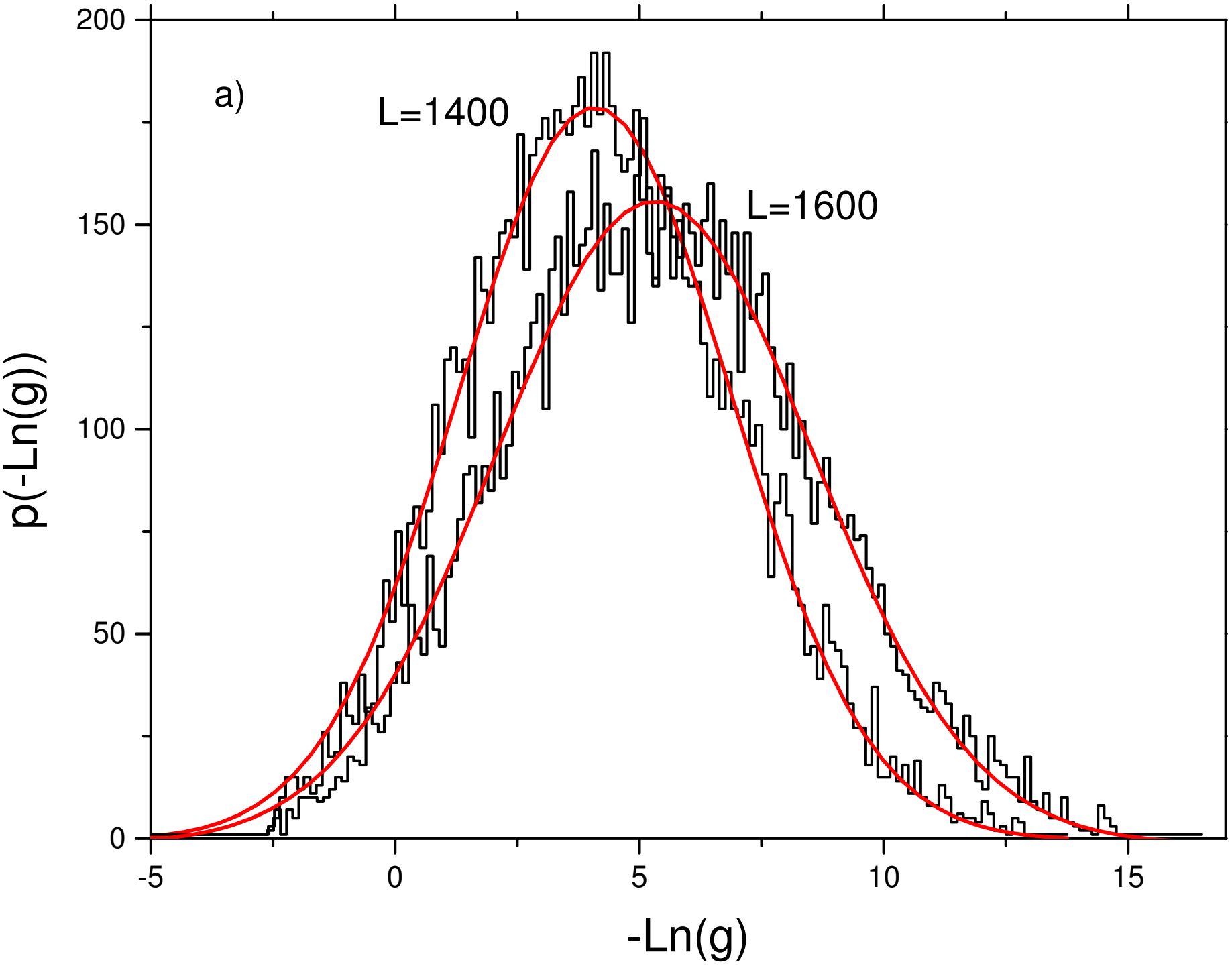


Figure 2b

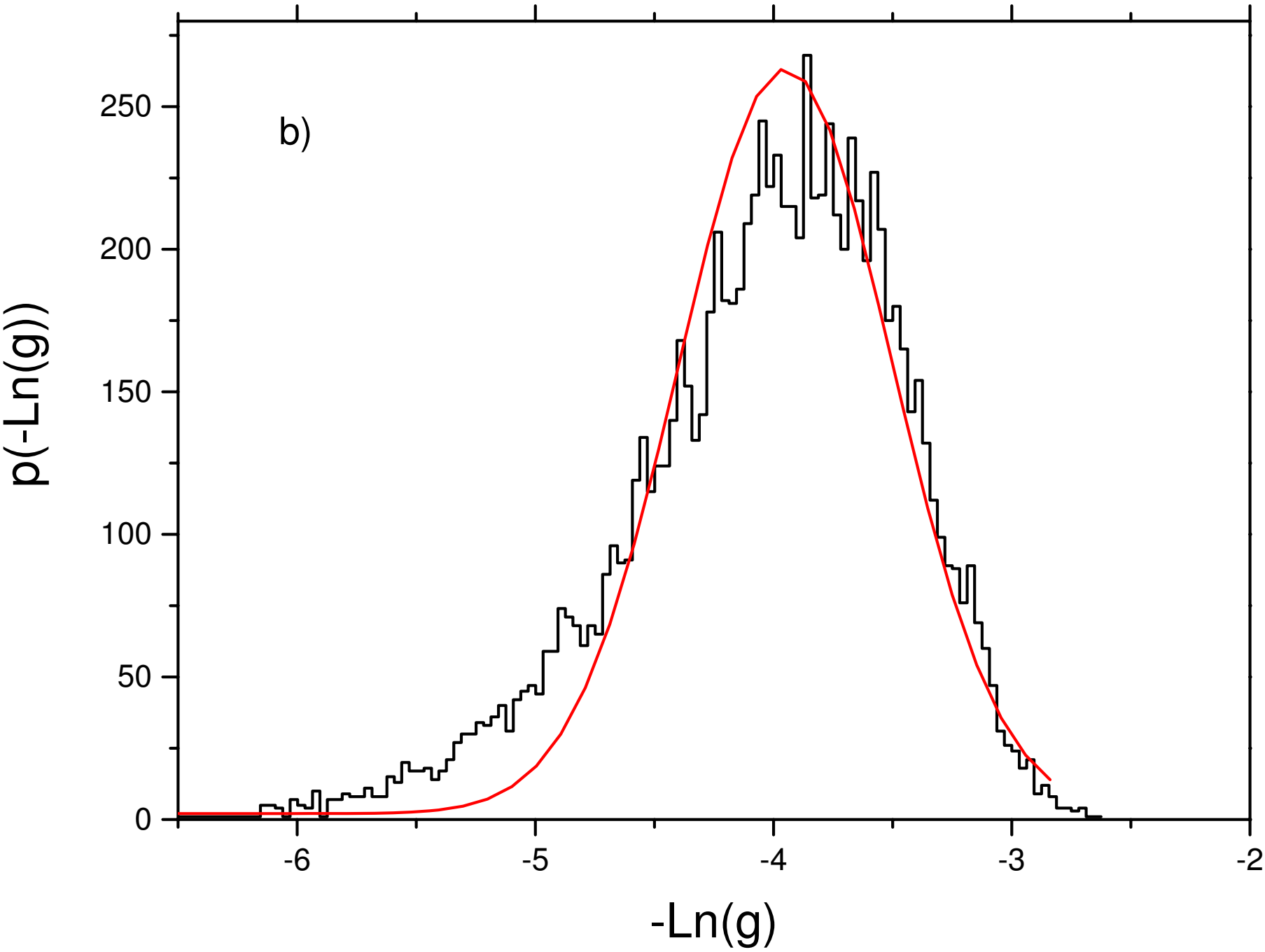
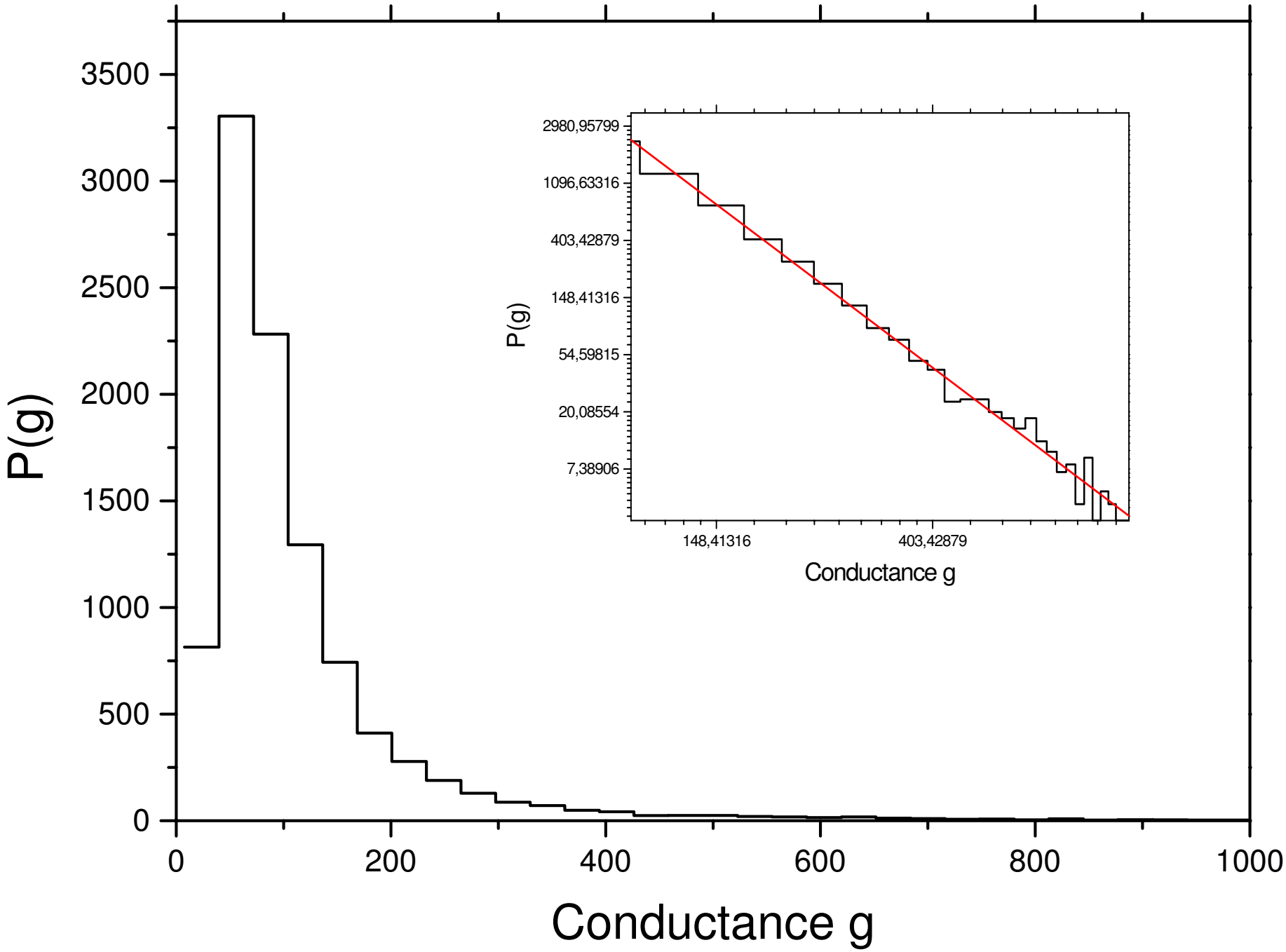


Figure 3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800
1000
Conductance g
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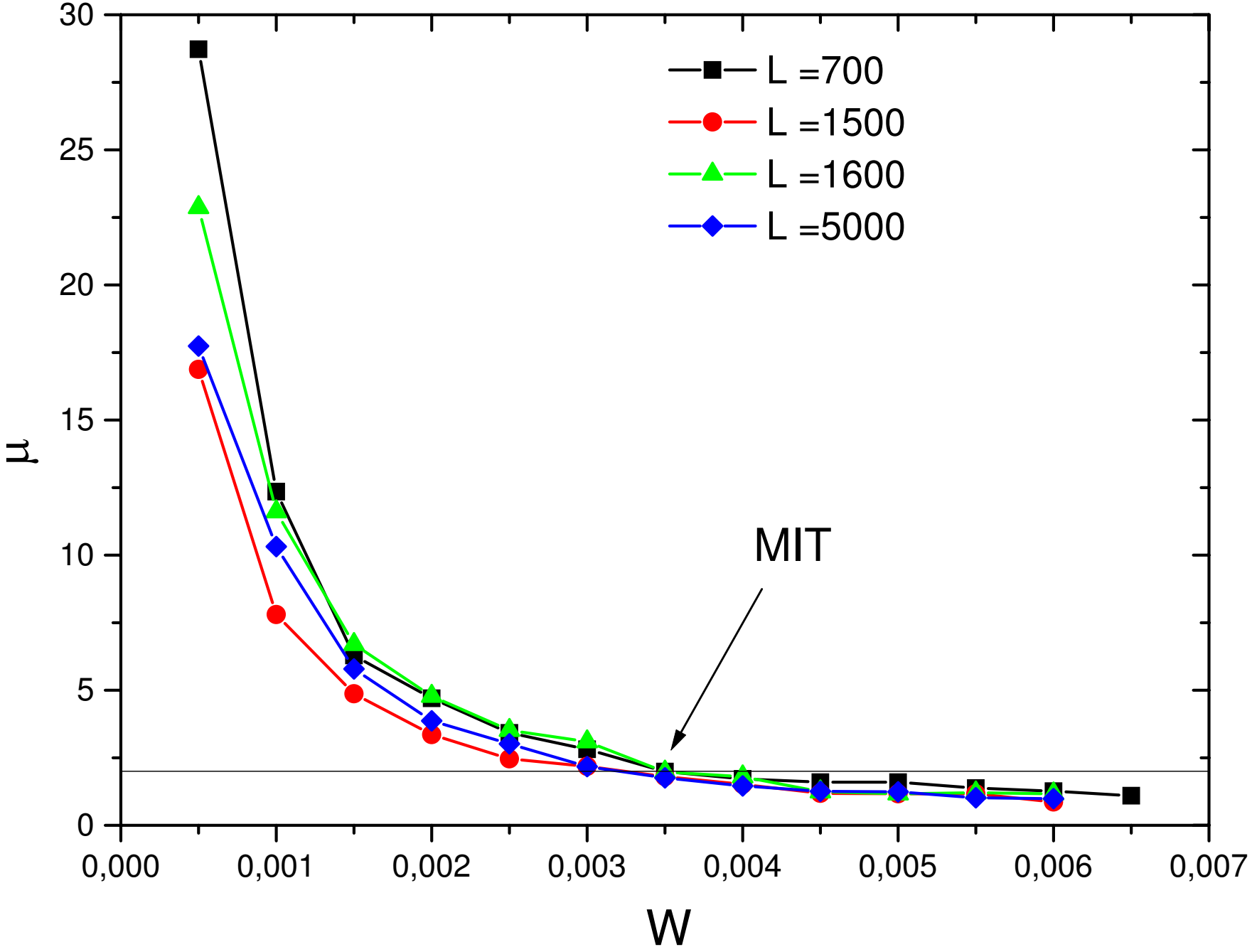
Conductance g

Figure 4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W
L =700
L =1500
L =1600
L =5000
MIT

Figure 5a

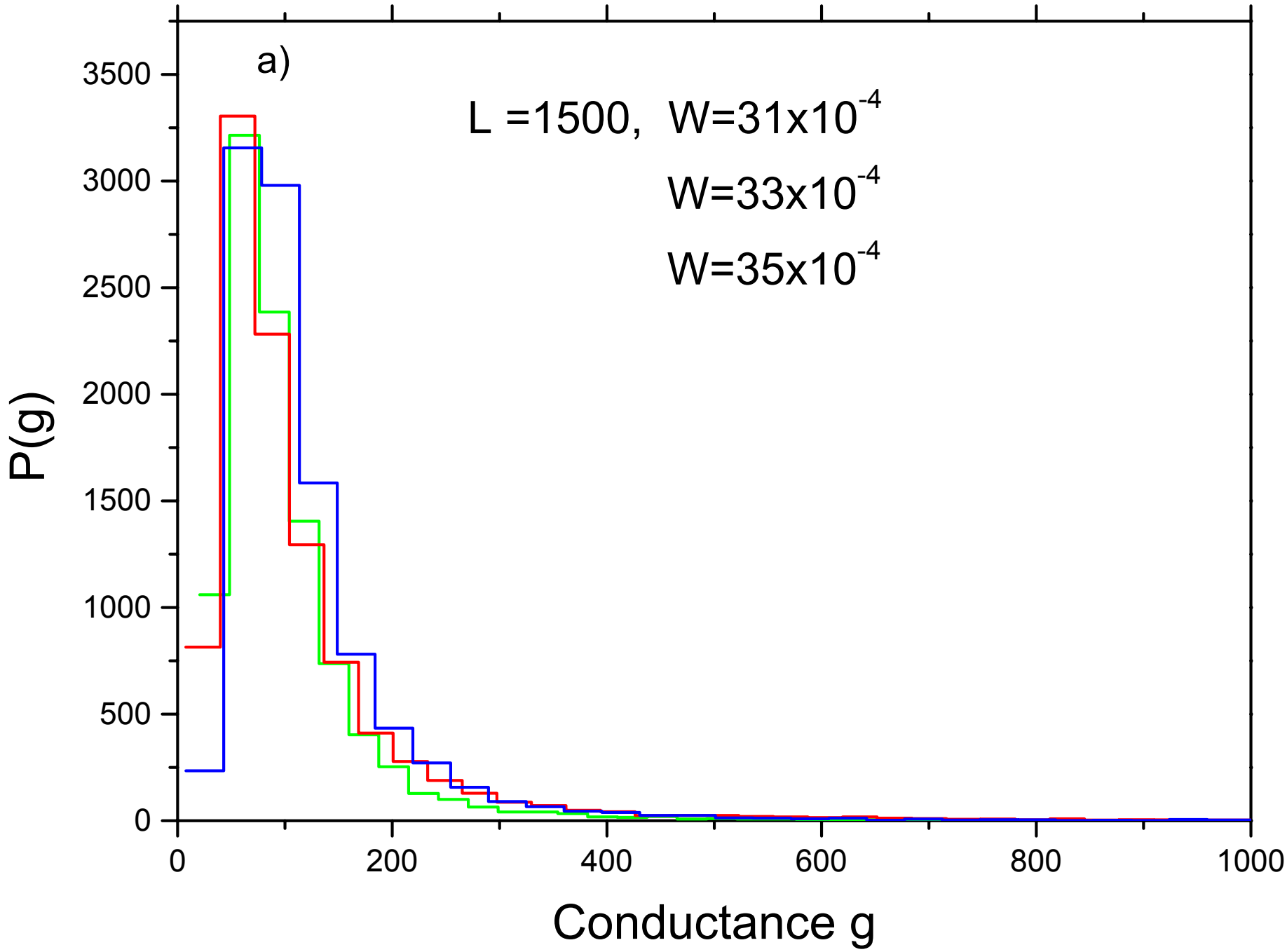
a)
L =1500, W=31x10-4
W=33x10-4
W=35x10-4
P(g)
Conductance g
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Figure 5b

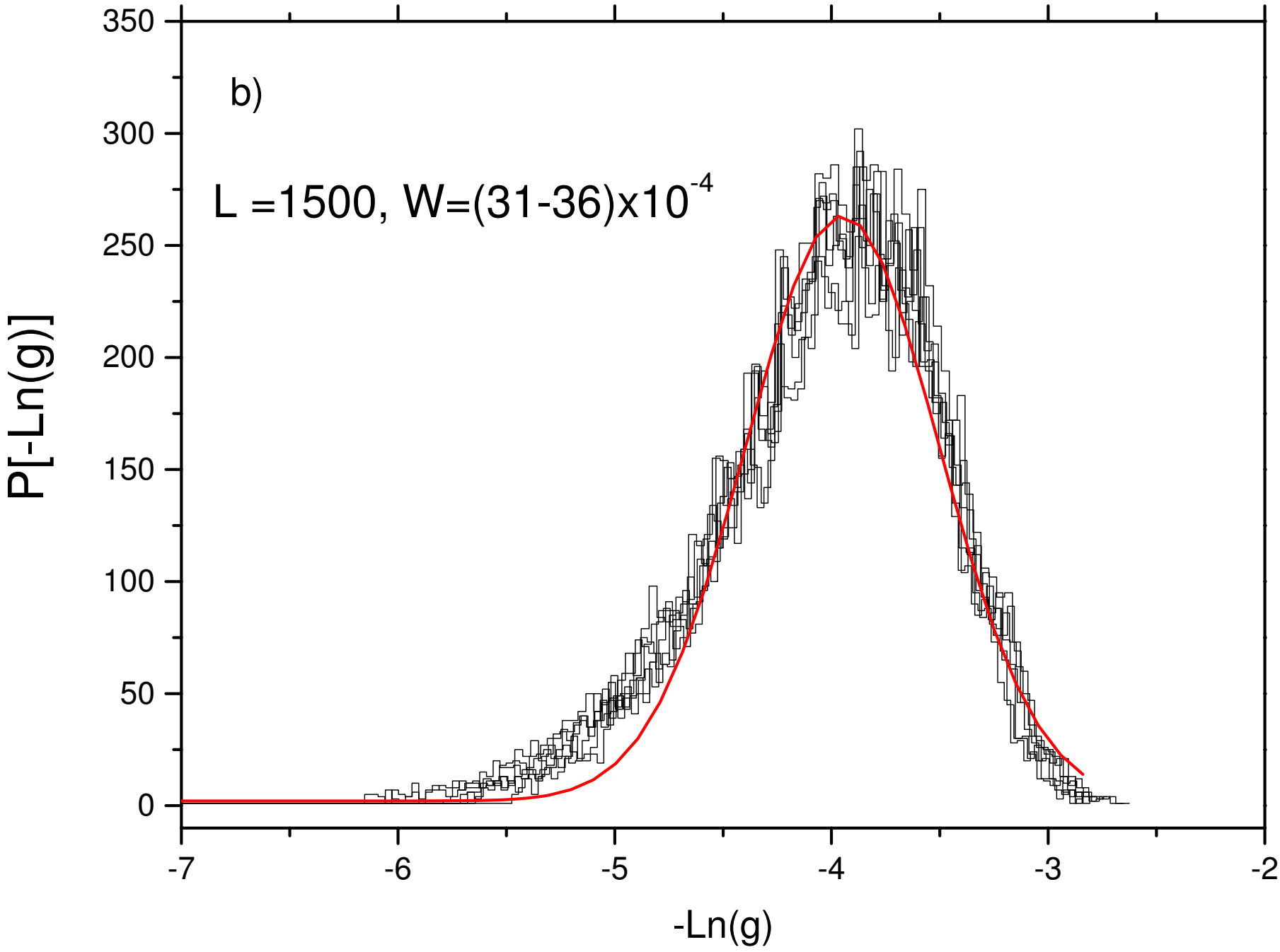
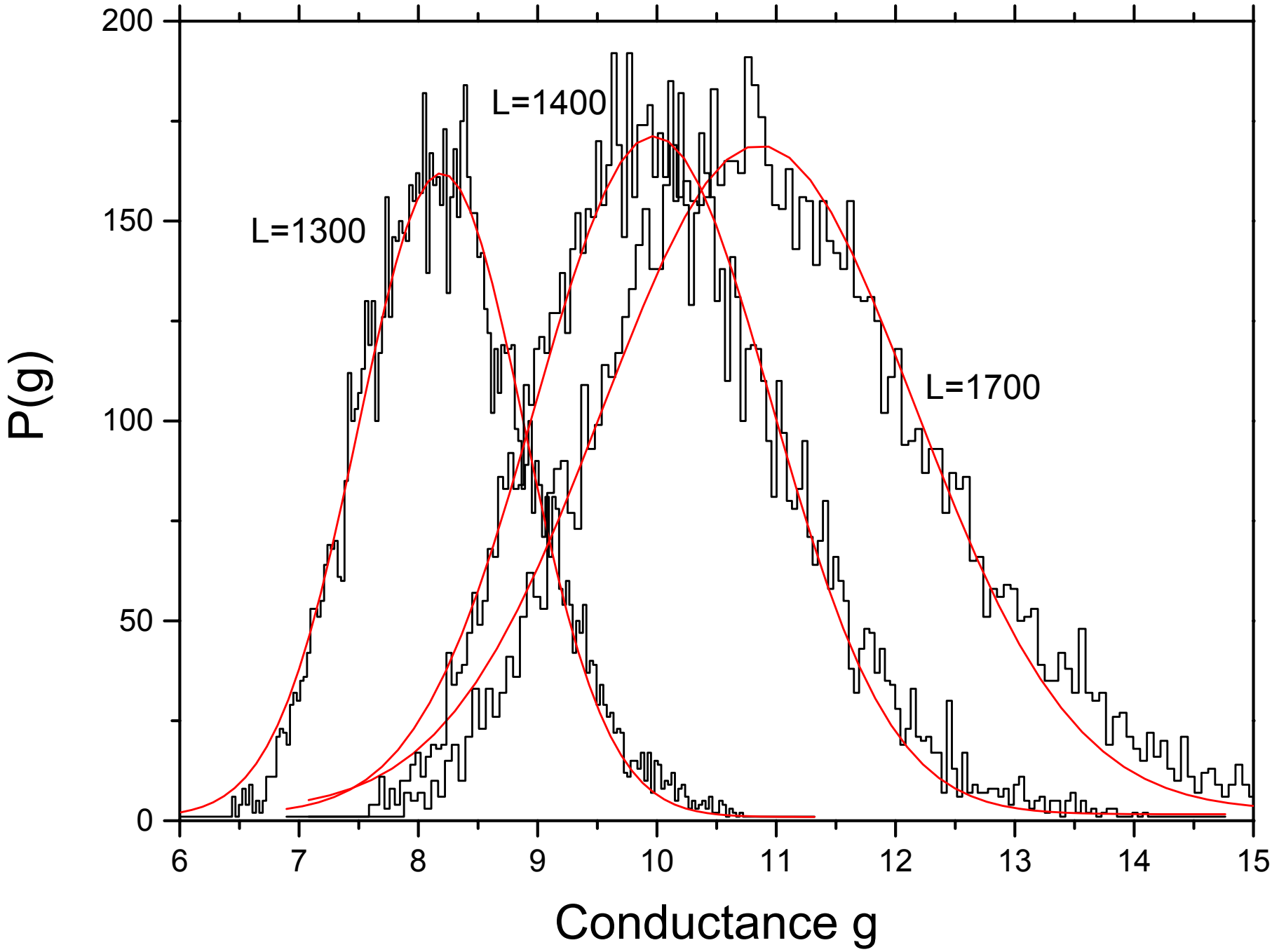


Figure 6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P(g)
L=1300
L=1400
L=1700
6
7
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12
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14
15
Conductance g